Inventories and Market Power in the World Crude Oil Market

by

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Executive Summary

This paper examines the role of inventories in determining market power in world crude oil markets with a simultaneous model of prices, stocks, production, and consumption of crude oil. The model considers the Organization for Petroleum Exporting Countries (OPEC) as a dominant firm with a competitive fringe. The demand side includes models for the consumption of crude and partial adjustment models for inventories. User costs are a function of prices for first and second nearby futures contracts, which are related via standard financial arbitrage models. The supply relation for OPEC depends upon the inverse demand elasticity for stocks and flows of crude and the supply elasticity from the competitive fringe. The complete structural specification allows a comparison of limited information estimators employing instrumental variables estimation with full information maximum likelihood. While the parameter estimates are similar, the normality assumption of the former seems problematic. The estimates indicate that mark-ups decline with higher beginning inventories. Model simulations reveal that markups are more sensitive to demand shocks than to supply shocks. The simulation analysis indicates that gradual accumulation of stocks for the US Strategic Petroleum Reserve (SPR) would have minimal impacts on prices. SPR sales during supply disruptions, however, only modestly reduce prices and markups.

Introduction

In determining the size and timing of crude oil sales from the U.S. Strategic Petroleum Reserve (SPR), government managers must consider market conditions in assessing the impacts of their decisions. A supply reduction during a period of relatively low inventories and high prices may justify a substantial sale of crude from the stockpile. In contrast, the same supply shortfall with excess inventories could argue for a much smaller sale or no sale at all. For these reasons, SPR managers and policy analysts seek to understand the extent to which market balance should influence their sales and stocking decisions.

The availability of crude oil in the world during any period equals the sum of current crude oil production and beginning inventories. Similarly, the demand for crude involves crude oil consumed during refinery distillation runs and crude oil stocked for next period. Given time lags in the transportation of oil from producing to consuming regions, significant costs associated with changing oil production rates in new fields, and production quotas by the Organization of Petroleum Exporting Countries (OPEC), inventories are the critical buffer balancing the market as it adjusts to supply and demand shocks. For this reason, producers and consumers closely follow inventory levels, presumably under the belief that stocks affect market prices. This paper seeks to determine whether there is any evidence supporting this intuition.

Our means to achieve this objective involves the specification and estimation of a short-run model of the world crude oil market. Numerous studies of crude oil markets find evidence supporting successful cartel pricing by OPEC, including those by Adelman (1982), Teece (1982), Griffin (1985), and Allhaji and Huettner (2000). None of these studies, however, considers short-run price determination and the role of inventories in market clearing and OPEC production decisions. This paper attempts to fill this void with a monthly model of world crude oil markets that simultaneously determines prices, inventories, production, and demand within an imperfectly competitive market structure.

The structure of the model essentially involves the supply and demand for flows and stocks of crude oil. The demand side recognizes that crude oil consumption or really flows into refinery distillation units are derived from the demand for refined petroleum products. Likewise, production flows of crude result from profit maximizing decisions by OPEC and non-OPEC producers. These production decisions, contingent upon stock levels, determine prices. Based upon the recent evidence by Allhaji and Huettner (2000), this study assumes OPEC acts as a dominant firm, pricing off their residual demand curve, which reflects supply from the competitive fringe and market demand for flows and stocks of crude oil.

The demand for crude oil stocks depends upon demand and supply shifters that vary depending upon the stage of processing and user costs that contain the financial opportunity cost of capital to finance the inventory and capital gains or losses associated with these inventory investments. The empirical commodity storage literature represented by Working (1934), Brennan (1958), Telser (1958), and Fama and French (1987) use spreads or differences in prices for futures contracts to approximate these capital gains or returns from storage. These studies posit a supply-of-storage relation that equates the returns to storage with a convenience yield earned from holding inventories, arising from production or cost smoothing benefits discussed by Blinder and Maccini (1991). Brennan, Williams, and Wright (1997) note that transportation costs and capacity constraints also may account for these inter-temporal price spreads. Our storage relation also includes a risk premium along the lines developed by Brennan and Schwartz [1985] and Gibson and Schwartz [1990]. The supply-of-storage relation closes the storage market equilibrium, essentially providing a solution for prices on the next nearby futures contract.

In summary, the model contains behavioral relations for the demand for crude oil, month-ending inventories, the supply-of-storage relation that provides the second nearby futures price, non-OPEC production, an OPEC supply relation for the first nearby futures price, and the market clearing identity solved for OPEC production. This formulation allows estimation with full information maximum likelihood, which provides a basis for comparison with instrumental variable estimation, such as three-stage least squares and generalized methods of moments, commonly employed in many inventory studies.

The next section presents the formulation of the model. The third section provides an overview of the sample and recent trends in stocks, production, demand, and prices in world crude oil markets. Econometric estimation and related issues appear in the fourth section of the paper. In the fifth section, the model is used to estimate the market impacts from a 30-million-barrel stock addition to the strategic petroleum reserve (SPR) and from a 1 million barrel per day supply disruption. To provide some perspective on the relative importance of supply and demand shocks to the world crude oil market, a third simulation of a 1 million barrel per day demand shock also is examined. The paper concludes with a summary of findings, a discussion of policy implications, and recommendations for future research.

The Model

Once crude oil is extracted from the earth, it is collected by pipelines at the oil field and then shipped either by pipeline or tanker to refineries where it is transformed into an array of petroleum products—including gasoline, distillate fuel, residual fuel oil, jet fuel, and petrochemical products. The demand for crude oil is derived from the demand for these refined petroleum products. Sales for these products have pronounced seasonality with gasoline sales rising during the summer and distillate sales increasing during the fall and winter in the northern hemisphere. As a result, there are corresponding seasonal swings in crude oil demand.

On the basis of data availability, this study considers the demand for crude oil in primary markets defined to include the USA, Japan, and fifteen European countries (EU15)[1]. Net requirements for the rest of the world, which include demand and net stock changes, are defined as the difference between world availability and consumption in the primary markets. Our previous observations about the nature of crude oil demand suggest that the demand for crude oil in primary markets, Q_t, is a function of real price and refined petroleum production:[2]

$$Q_t = \alpha_0 + \alpha_1 \ln\left(P_t / CPI_t\right) + \alpha_2 \ln Z_t + \varepsilon_t^Q, \qquad \varepsilon_t = \rho_Q \varepsilon_{t-1}^Q + v_t^Q, \tag{1}$$

where P_t is the price on the first nearby contract at the New York Mercantile Exchange (NYMEX) for West Texas Intermediate (WTI) crude oil, CPI_t is the U.S. consumer price

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[1] The fifteen countries are Austria, Belguim, Denmark, Finland, France, Germany, Greece, Ireland, Italy ,Luxemburg, Netherlands, Portugal, Spain, Sweden, United Kingdom

[2] Note that there are separate balances for crude oil and refined petroleum products. Refined petroleum product production by definition equals shipments of refined products plus ending stocks of products less beginning inventories of products. The crude oil balance is totally separate from the product balance.

index, and Z_t is production of refined petroleum products in period t. Notice that the error term is allowed to display first-order serial correlation. This relation notes that the demand for crude oil is derived from the demand for refined petroleum products, such as gasoline and distillate fuel oil. This equation has a semi-log specification because it allows a closed form solution for price and the inverse price elasticity that enters the supply relation.

The NYMEX price for WTI crude is the most actively traded crude oil futures contract in the world and is widely regarded as the market-clearing price, from which many other crude oil types of varying sulfur and gravity are priced. Moreover, most spot transactions are guided by the NYMEX WTI price. Prices on spot transactions are collected by telephone surveys of traders but these tend to be less reliable than exchange reported prices. Moreover, Considine and Larson (2001) found that the first nearby futures price is highly correlated with the price for spot or cash transactions and that parameter estimates for supply of storage relations are very close for both sets of prices.

Given the unavailability of data on refined petroleum production outside primary markets, net requirements, R_t, in the rest of world is a function of real price and a simple time trend, T, as follows:

$$R_t = \beta_0 + \beta_1 \ln\left(P_t / CPI_t\right) + \beta_2 T_t + \varepsilon_t^R, \qquad \varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + v_t^R \tag{2}$$

Apart from market demand shocks, such as the Asian financial crisis of 1998, the use of trend in this equation serves as a proxy for steady economic growth in emerging market economies, which generates much of the growth in crude oil consumption for the rest of the world.

Inventories of crude are held at various points in the petroleum distribution network. Holders of oil leases, or field producers, hold a small amount in storage tanks at producing fields. The largest private stocks on land — nearly two-thirds of total private stocks in the United States — are in pipelines and bulk storage facilities. The remaining third of private stocks is held at petroleum refineries. Significant stocks of crude are on oil tankers at sea. Governments in the U.S., Japan, and the European Union also hold stocks of crude oil for strategic use in the event of emergencies. Changes in total stocks, however, in any given month are dominated by swings in private commercial stock holding on land and at sea.

To ensure a stable, closed-form expression for inventories, this study adopts a simple partial-adjustment model of crude oil inventories in primary markets. Commercial stocks in primary markets, X_t, depend upon real user costs, refinery production, and lagged stocks:

$$X_t = \delta_0 + \delta_1 \left[\ln\left(P_t / P_t^f \right) + r_t - i_t \right] + \delta_2 \ln Z_t + \delta_3 X_{t-1} + v_t^X , \tag{3}$$

where the first expression in square brackets on the right of (3) is real user cost; P_t^f is the price on the second nearby NYMEX contract for WTI crude oil; r_t is the monthly rate on three month U.S. Treasury bills; and i_t is the monthly rate of change in the U.S. consumer price index. The expectation is that stocks fall with higher user costs and decline with higher refinery production, which reflects production smoothing by refiners and transporters of crude oil. The relation for crude oil stocks at sea, S_t, follows a similar specification:

$$S_t = \gamma_0 + \gamma_1 \left[\ln\left(P_t / P_t^f \right) + r_t - i_t \right] + \gamma_2 \ln\left(Y_t^o + Y_t^{no} \right) + \gamma_3 S_{t-1} + v_t^S, \qquad (4)$$

where Y_t^o is OPEC production, and Y_t^{no} is non-OPEC production. Again, our expectation is that stocks at sea should fall with higher user costs. Unlike inventories on land, stocks at sea, or floating inventories depend upon world production. Given the time lags in transmitting demand shocks back to producers and in shipping crude from distant producing areas to the main consuming regions, stocks at sea tend to move directly with world oil production. Hence, the expectation here is that stocks at sea rise (fall) with higher (lower) world oil production. The two flow demand and two stock demand equations constitute the demand side of the model.

The next four equations represent the supply side of the market. The supply side of the storage market states that the nominal returns from storage reflect a marginal convenience yield and a risk premium:

$$\ln\left(P_t^f / P_t \right) - r_t = \phi_0 + \phi_1 \ln\left(X_{t-1} / Q_t \right) + \phi_2 \ln V_t \ln\left(P_t / CPI_t \right) \qquad \varepsilon_t^f = \rho_f \varepsilon_{t-1}^f + v_t^f \qquad (5)$$

The first two terms on the right represent the convenience yield, which is expected to be increasing in the inventory-to-sales ratio so that at low stock levels the return to storage could be very small or negative. Based upon the study by Considine and Larson (2001), crude oil price backwardations, or when prices on the first nearby contract exceed those on the second nearby contract, occur during periods of high price volatility. This suggests that the returns to storage fall with higher price volatility, which is measured here as a 20-day moving average of the standard deviation in the price for the first nearby contract.

We now consider the supply for crude oil from the world oil cartel, OPEC, and non-OPEC producers that may or may not collude with the cartel. Marginal revenue facing OPEC is:

$$\frac{\partial RV_t^o}{\partial Y_t^o} = P_t + Y_t^o \frac{\partial P_t}{\partial Y_t^o}, \tag{6}$$

where Y_t^o is crude production by OPEC in period t. Equating marginal revenue with marginal cost and solving for OPEC output yields:

$$P_t = MC_t^o - \frac{\partial P_t}{\partial \ln Y_t^o} \tag{7}$$

Marginal cost is approximated using the procedures discussed by Allhaji and Huettner (2001) among other studies. Marginal cost consists of incremental extraction cost of roughly $4 in 1980, which is escalated at the rate of inflation in this study, and royalty payments, which are on average 17% of price for OPEC producers. Equilibrium OPEC output is a residual demand:

$$Y_t^o = Q_t + R_t - Y_t^{no} + \left[\left(X_t - X_{t-1} \right) + \left(S_t - S_{t-1} \right) + \left(G_t - G_{t-1} \right) \right] / D_t, \tag{8}$$

where G_t denotes government held stocks of crude oil, and D_t is the number of days per month.

Non-OPEC production, Y_t^{no} depends upon the real market price and OPEC production:

$$Y_t^{no} = \pi_0 + \pi_1 \ln\left(P_t / CPI_t \right) + \pi_2 \ln Y_t^o + \varepsilon_t^{no}, \quad \varepsilon_t^{no} = \rho_{no} \varepsilon_{t-1}^{no} + v_t^{no}. \tag{9}$$

Non-OPEC production would likely increase with real prices and could increase or decrease with OPEC production depending upon the extent of collusion between the two sectors.

We now have all the elements to derive the derivative in equation (7). The first step is to note that by the returns to storage relation, second nearby futures prices are affected by the first nearby prices. Hence, futures prices from the two inventory demand equations must be eliminated by substituting (5) into (3) and (4) to obtain:

$$\begin{aligned} X_t &= \delta_0 - \delta_1 \left[\phi_0 + \phi_1 \ln\left(X_{t-1}/Q_t \right) + \phi_2 \ln V_t \ln\left(P_t/CPI_t \right) + i_t + \varepsilon_t^f \right] \\ &\quad + \delta_2 \ln Z_t + \delta_3 X_{t-1} \end{aligned} \tag{10}$$

$$\begin{aligned} S_t &= \gamma_0 - \gamma_1 \left[\phi_0 + \phi_1 \ln\left(X_{t-1}/Q_t \right) + \phi_2 \ln V_t \ln\left(P_t/CPI_t \right) + i_t + \varepsilon_t^f \right] \\ &\quad + \gamma_2 \ln\left(Y_t^o + Y_t^{no} \right) + \gamma_3 S_{t-1} \end{aligned}. \tag{11}$$

The demand for OPEC crude follows from substituting (1), (2), (9), (10) and (11) into (8) resulting in:

$$\begin{aligned} Y_t^o &= \alpha_0 + \alpha_1 \ln\left(P_t/CPI_t \right) + \alpha_2 \ln Z_t + \beta_0 + \beta_1 \ln\left(P_t/CPI_t \right) + \beta_2 T_t \\ &\quad - \left[\pi_0 + \pi_1 \ln\left(P_t/CPI_t \right) + \pi_2 \ln Y_t^o \right] - \left[X_{t-1} + S_{t-1} - \left(G_t - G_{t-1} \right) \right] \big/ D_t \\ &\quad + \left\{ \delta_0 - \delta_1 \left[\phi_0 + \phi_1 \ln\left(X_{t-1}/Q_t \right) + \phi_2 \ln V_t \ln\left(P_t/CPI_t \right) + i_t \right] \right. \\ &\quad \left. + \delta_2 \ln Z_t + \delta_3 X_{t-1} \right\} \big/ D_t \\ &\quad + \left\{ \gamma_0 - \gamma_1 \left[\phi_0 + \phi_1 \ln\left(X_{t-1}/Q_t \right) + \phi_2 \ln V_t \ln\left(P_t/CPI_t \right) + i_t \right] \right. \\ &\quad \left. + \gamma_2 \ln\left(Y_t^o + Y_t^{no} \right) + \gamma_3 S_{t-1} \right\} \big/ D_t + \varepsilon_{t-1} + v_t , \end{aligned} \tag{12}$$

where

$$\begin{aligned} \varepsilon_{t-1} &= \rho_Q \varepsilon_{t-1}^Q + \rho_R \varepsilon_{t-1}^R - \rho_{no} \varepsilon_{t-1}^{no} - \left(\delta_1 + \gamma_1 \right) \rho_f \varepsilon_{t-1}^f \big/ D_t \\ v_t &= v_t^Q + v_t^R v_t^{no} + \left(v_t^S + v_t^X \right) \big/ D_t - \left(\delta_1 + \gamma_1 \right) v_t^f \big/ D_t . \end{aligned}$$

Solving for P_t yields[3]:

$$P_t^e = \exp\begin{bmatrix} \{-\mu_0 - (\alpha_2 + \delta_2/D_t)\ln Z_t - \beta_2 T + \pi_2 \ln Y_t^o \\ +Y_t^o - \gamma_2 \ln(Y_t^o + Y_t^{no})/D_t + \phi_1(\delta_1 + \gamma_1)\ln(X_{t-1}/Q_t)/D_t \\ +(\delta_1+\gamma_1)i_t/D_t + \left[(1-\delta_3)X_{t-1} + (1-\lambda_3)S_{t-1} - (G_t - G_{t-1})\right]/D_t \\ -\varepsilon_{t-1} - v_t\}/\mu_1 - \ln CPI_t \end{bmatrix} \quad (13)$$

where

$$\mu_0 = \alpha_0 + \beta_0 - \pi_0 + \left[(\delta_0 + \gamma_0) - \phi_0(\delta_1 + \gamma_1)\right]/D_t \quad (14)$$

$$\mu_1 = \alpha_1 + \beta_1 - \pi_1 - \phi_2(\delta_1 + \gamma_1)\ln V_t/D_t \quad (15)$$

Taking the partial derivative of (13) with respect to OPEC output and substituting into (7) provides an equilibrium expression for the markup:

$$P_t - MC_t^o = \frac{-\left\{Y_t^o + \pi_2 - \gamma_2\left[Y_t^o/(Y_t^o + Y_t^{no})\right]/D_t\right\}P_t^e}{\mu_1} \quad (16)$$

For estimation, the autoregressive terms from the demand side and non-OPEC production must be substituted into (16)[4]. Solving for prices and assuming a first-order autoregressive process provides the following empirical model for the supply relation:

$$P_t = MC_t^o - \frac{\left\{Y_t^o + \pi_2 - \gamma_2\left[Y_t^o/(Y_t^o + Y_t^{no})\right]/D_t\right\}P_t^e}{\mu_1} + \varepsilon_t^p, \quad \varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + v_t^p \quad (17)$$

This behavioral condition provides the equilibrium price for OPEC that maximizes their profits subject to their residual demand curve that reflects the flow and stock demand for crude and supply from non-OPEC producers. The complete econometric model consists

[3] Details appear in Appendix A.
[4] See Appenidx A.

of seven behavioral equations (1)-(5), (9), and (17), and one identity (8) containing eight endogenous variables, Q_t, R_t, X_t, S_t, Y_t^{no}, Y_t^o, P_t, and P_t^f.

World Oil Markets

Our sample period is from January 1987 to December 2000[5]. Summary statistics on crude stocks and flows appear in Table 1. World production averaged 67.1 million barrels per day (mmbd). Key reporting areas consumed 28.6 mmbd. The mean for net requirements for the rest of the world is 39 million barrels per day. A time series plot of consumption and net requirements appears in Figure 2. Net requirements in the rest of the world appears reasonable in light of consumption trends in key reporting areas, picking up the sharp drop in consumption related to the Asian financial crisis of 1998.

OPEC production averaged 26.6 million barrels per day (mmbd) over the sample period, rising from slightly over 15 mmbd in early 1987 to over 30 mmbd by late 1997. OPEC output stagnated during 1998 and 1999 but then began to rise to slightly over 32 mmdb by October 2000. Non-OPEC production averaged 41.2 mmbd, drifting from nearly 42 mmbd in early 1987 to under 39 mmbd during 1993 but then increased to nearly 45 mmbd by the end of 2000.

Commercial inventories of crude oil held in primary markets averaged 798.6 million barrels. One surprising aspect is that inventories at sea are as large as commercial stocks held in the key reporting areas. Figure 3 indicates that inventories in the key reporting areas hovered around 798 million barrels with a slight downward trend during the last three years. Stocks held at sea remained below inventories in the key reporting areas until 1991 but then consistently exceeded them since 1996.

[5] Appendix B provides a summary of the data sources.

Table 1: Summary Statistics on World Crude Oil Markets, 1987 to 2001

	Mean	Standard Deviation	Minimum	Maximum
	Million barrels per day			
Sales, primary markets	28.6	2.4	23.0	32.8
Net requirements, rest of the world	39.0	2.3	32.4	45.2
World Production	67.8	4.2	57.1	76.2
OPEC	26.6	3.5	15.9	32.3
Non-OPEC	41.2	1.4	38.8	45.0
	Million barrels			
Primary Markets	798.6	28.9	731.0	901.0
At Sea	802.4	48.1	657.0	892.0
Government	1,217.8	96.6	910.0	1,318.0
	Dollars per barrel			
First nearby	20.1	4.5	11.3	35.9
Second nearby	19.9	4.2	11.6	35.0
	Percent per annum			
Returns to storage	-14.4	26.3	-117.1	64.2
Real user costs	11.2	25.7	-65.6	112.5

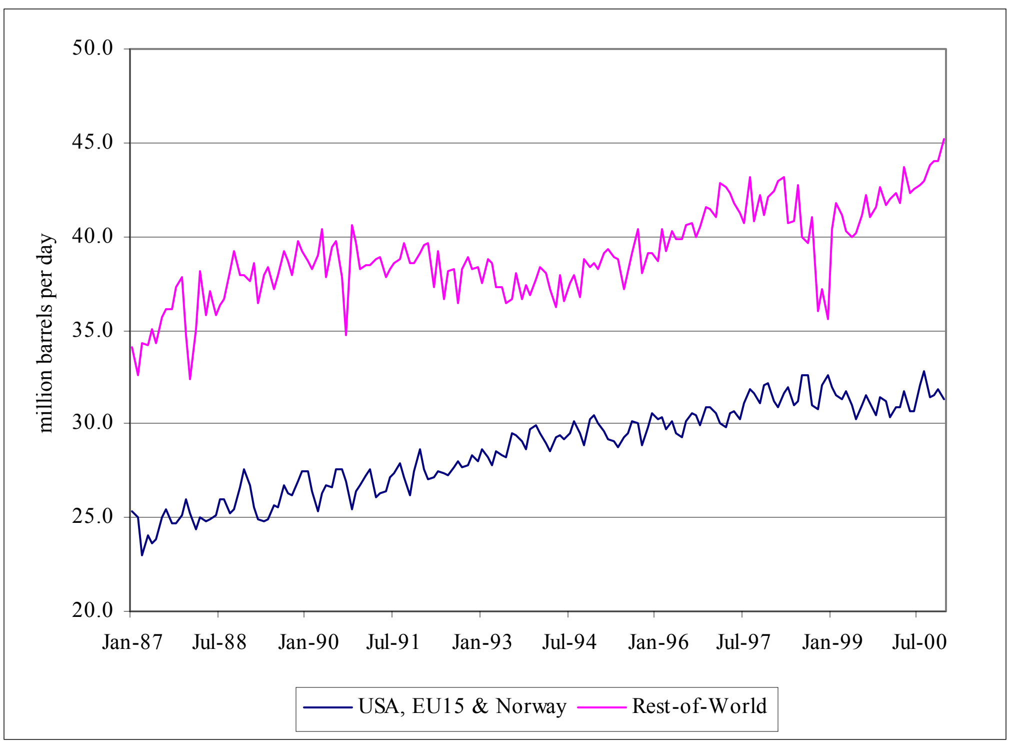

Figure 1: Regional consumption and net requirements of crude oil from 1987 to 2001

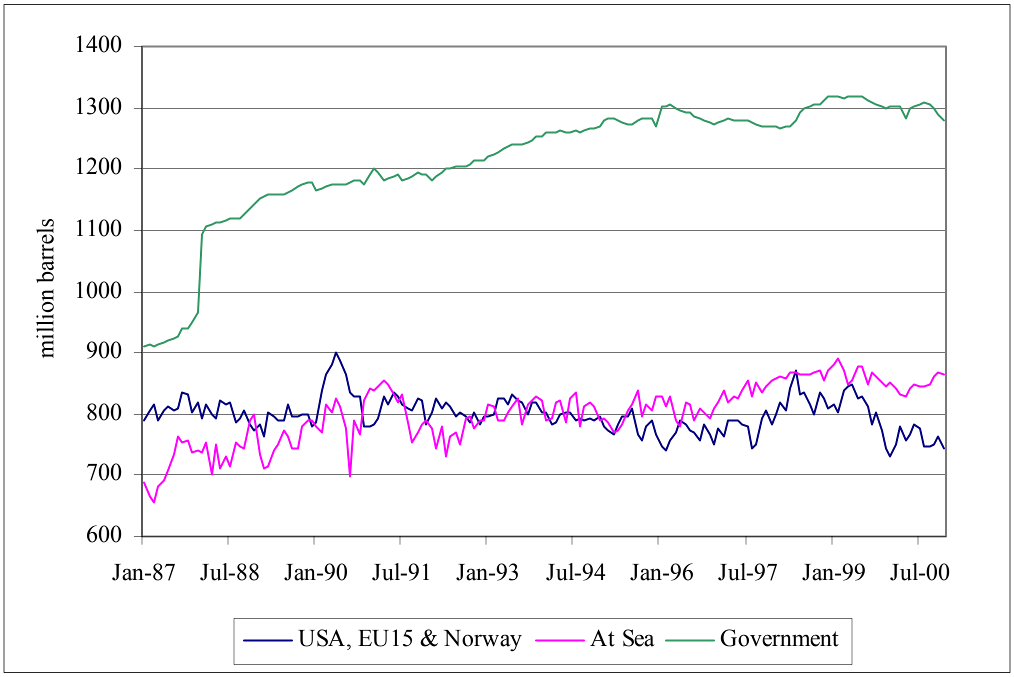

Figure 2: Commercial and government stocks of crude oil

Prices for the first nearby contract averaged $20 per barrel over the sample period, ranging from a high of almost $36 during October 1990 to below $12 during late 1998. A plot of prices and the inventory-to-sales ratio in primary markets, or days of supply, appear in Figure 3. Days of supply fell from nearly 35 from 1987 to under 25 by late 2000, illustrating a trend in the industry to more efficient inventory management systems. The relationship between prices and days of supply appears quite erratic, with a rather weak 0.18 simple correlation.

Returns to storage expressed as an annual rate of return displays enormous volatility ranging from –117% to over 64% (see Table 1). The mean return to crude oil storage is negative, suggesting that on average the crude oil market was in backwardation. Considine and Larson (2001) find that the principle factor explaining this backwardation is price volatility. Measures of relative stock availability are relatively

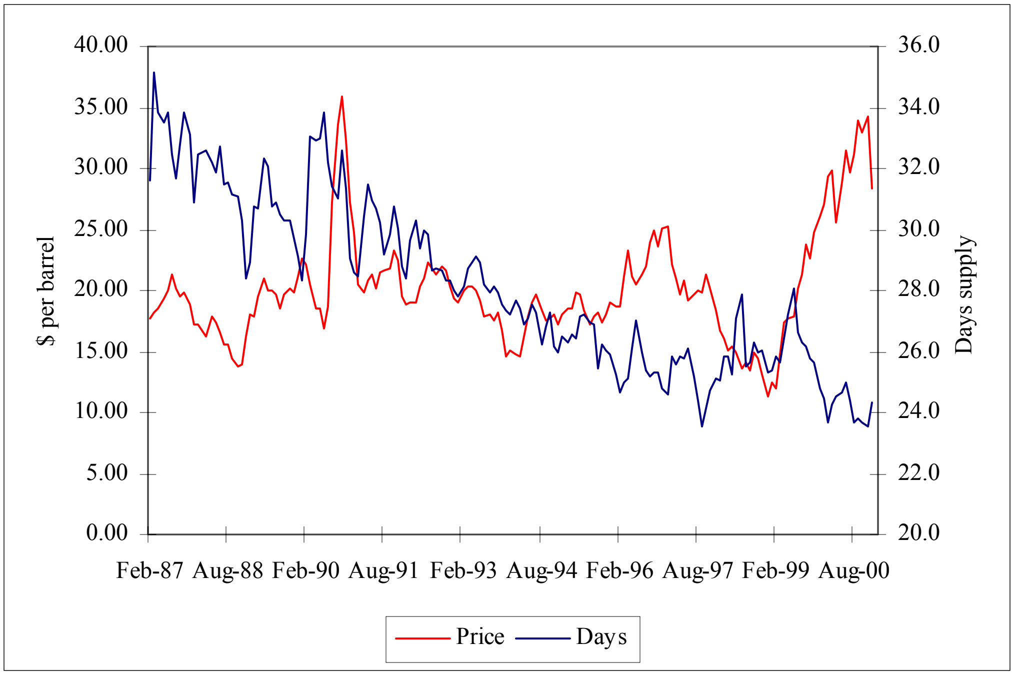

Figure 3: Crude oil prices and days supply

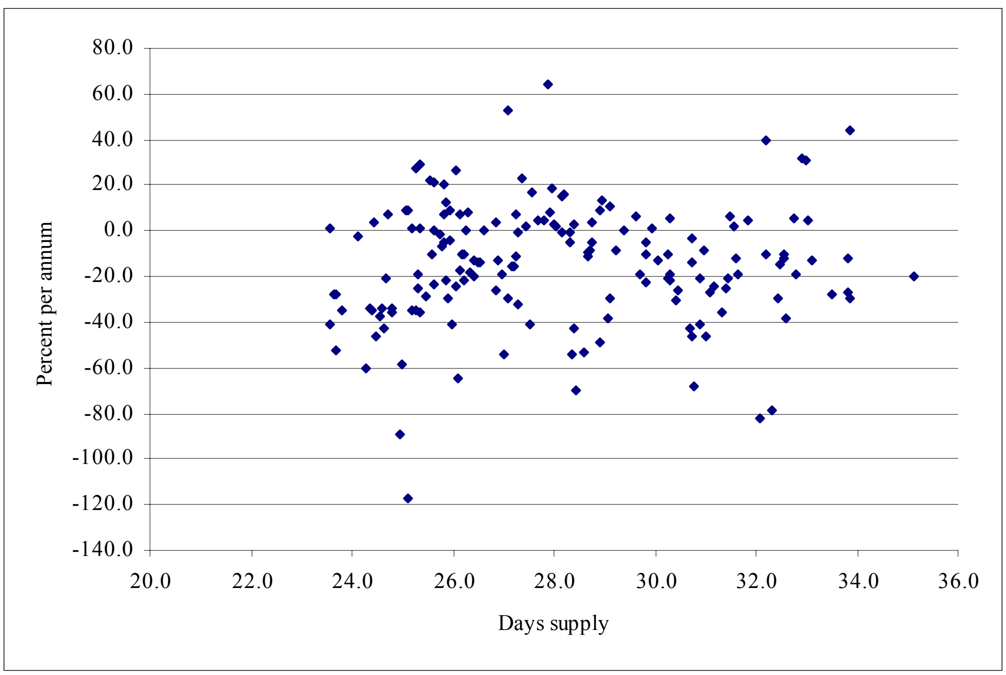

Figure 4: Returns to storage and days supply

unimportant. A plot of returns to storage and days of supply displayed in Figure 4 also does not convincingly display the supply-of-storage relation. For the classic supply of storage to hold, negative returns should occur at low inventory to sales levels and large positive returns should be associated with relatively high stock levels. This cursory examination of the data suggests that there is no simple, clear-cut relationship between days of crude oil supply, prices, or returns to storage.

Econometric Estimates

Unlike many inventory studies, the simultaneous model of prices, stocks, and flows in this study allows estimation with full information maximum likelihood (FIML). Sensitivity of estimates to instrumental variable selection is thereby avoided. On the other hand, the normality assumption of FIML may not be appropriate. Accordingly, this paper also estimates the model using two instrumental variable estimators: three-stage least squares (3SLS) with White's correction of standard errors for heteroscedasticity, and generalized method of moments (GMM) with a heteroscedastic consistent error weighting matrix. The instruments include monthly dummy variables and lagged values of land, sea, and government crude oil inventories, OPEC and non-OPEC production, refinery production, first and second nearby prices, rates on the three-month Treasury bill, the consumer price index, and price volatility. As presented in the previous section, all equations are corrected for first-order autocorrelation using quasi-first differencing, except the partial adjustment equations for inventories.

The GMM, 3SLS, and GMM parameter estimates appear in Table 2. The GMM and 3SLS estimates all have the expected sign. The FIML estimates are similar except that δ_2, or the coefficient of refinery production in the stock equation for primary

Table 2: GMM, 3SLS, and FIML parameter estimates.

	GMM		3SLS		FIML	
Coefficient	*Estimate*	*t-ratio*	*Estimate*	*t-ratio*	*Estimate*	*t-ratio*
α_0	29.034	10.022	26.900	4.068	28.545	0.593
α_1	-8.936	-15.318	-8.417	-5.492	-3.888	-0.156
α_2	6.869	8.439	7.005	3.599	3.055	0.528
ρ_O	0.965	81.178	0.957	24.584	0.906	1.773
β_0	66.306	13.372	62.451	3.436	112.798	1.582
β_1	-11.718	-12.335	-11.601	-4.206	-38.232	-7.065
β_2	0.045	1.463	0.071	0.657	0.224	0.288
ρ_R	0.977	62.846	0.975	23.438	0.914	8.175
δ_0	423.237	11.451	408.116	3.274	69.016	0.487
δ_1	-193.025	-7.675	-194.110	-2.110	-155.164	-0.668
δ_2	-55.071	-5.736	-50.047	-1.856	36.784	0.274
ρ_X	0.708	33.255	0.706	10.534	0.756	1.475
γ_0	-486.372	-9.257	-491.157	-3.728	-744.405	-3.751
γ_1	-78.762	-3.070	-101.078	-1.472	-90.341	-0.155
γ_2	174.543	10.953	176.482	4.405	257.729	4.643
ρ_S	0.692	33.761	0.688	11.570	0.577	9.293
π_0	75.188	25.000	74.240	9.507	47.381	0.753
π_1	3.164	7.865	3.435	2.596	-0.393	-0.106
π_2	-12.108	-13.796	-12.153	-5.364	-1.267	-0.084
ρ_{YNO}	0.967	156.089	0.966	56.020	0.982	17.448
ϕ_0	0.498	3.000	0.525	0.566	-0.336	-0.891
ϕ_1	0.007	0.157	-0.003	-0.011	0.111	0.935
ρ_f	0.863	55.251	0.854	14.164	0.824	16.533
ϕ_2	-0.062	-15.018	-0.061	-2.215	-0.004	-1.192
ρ_P	0.924	114.960	0.919	28.958	0.698	7.339
Overid test		151.087		275.206		
P-value		0.178		0.000		
Log-likelihood						-1683.3
Equation	*R-Squared*	*DW*	*R-Squared*	*DW*	*R-Squared*	*DW*
Q	0.890	1.878	0.871	1.908	0.914	1.926
R	0.610	2.405	0.616	2.408	0.374	1.407
X	0.638	1.995	0.640	1.988	0.644	1.890
S	0.808	2.384	0.808	2.375	0.814	2.146
YNO	0.805	1.968	0.800	1.943	0.900	2.350
P	0.901	2.283	0.900	2.278	0.902	1.561
PF	0.957	1.938	0.958	1.920	0.997	1.682

markets, is positive from the FIML estimation. The only other notable difference, at least in terms of sign, is the FIML estimate for π_1, the effect of price on non-OPEC production is negative, unlike the positive estimates under GMM and 3SLS. The fit of the equations as measured by the conventional R^2 is reasonably good for all estimation methods. The equation for net requirements for the rest of the world, however, is relatively poor for FIML. Overall, the t-ratios for the FIML estimates are substantially smaller than those estimated using GMM and 3SLS, which suggests that the normality assumption maybe problematic for this example. The 3SLS and GMM parameter estimates are very similar, except the standard errors for the former are substantially smaller than White's standard errors using 3SLS. This is not surprising because GMM weights each observation in inverse proportion to its variance, with a potentially substantial gain in efficiency.

The objective functions for 3SLS and GMM provide the basis for a test of the specification of the model. This test is distributed as a chi-squared statistic with degrees of freedom equal to the number of instruments multiplied by the number of equations less the number of parameters. The 3SLS test statistic is 275.2 far exceeding the one percent critical value of 177.3, suggesting that the over-identifying restrictions cannot be accepted. In contrast, the GMM test statistic is 151.1 with a probability value of 0.18, indicating that the restrictions cannot be rejected. These findings suggest that heteroscedastic weighting method of GMM substantially improves the efficiency of the estimates with little or no apparent effect on the parameter estimates. While FIML may be appealing in terms of eliminating sensitivity to instrument selection, the 3SLS and GMM estimates suggest that the normality assumption may be problematic. Hence, our subsequent discussion focuses on the GMM estimates.

The partial equilibrium elasticity estimates are presented in Table 3. First, the estimated short-run price elasticity of demand in primary markets is approximately -0.31 at the sample mean. The price elasticity of net requirements for the rest of the world is also very similar. The elasticities of crude oil inventory demand to user costs are also significant but very small (See Table 3). This small size, however, should be considered in light of substantial changes in user costs. In addition, given relatively high levels of stocks, a small percent change in the stock translates into a rather substantial change in

Table 3: GMM, 3SLS, and FIML partial equilibrium elasticity estimates.

	GMM		**3SLS**		**FIML**	
	Estimate	*t-ratio*	*Estimate*	*t-ratio*	*Estimate*	*t-ratio*
Primary markets						
Own price	-0.312	-15.32	-0.294	-5.49	-0.136	-0.16
Refinery Production	0.240	8.44	0.245	3.60	0.107	0.53
Rest of world net requirements						
Own price	-0.300	-12.33	-0.297	-4.21	-0.990	-7.06
Trend	0.001	1.46	0.002	0.66	0.006	0.29
Stocks primary markets						
User cost	-0.002	-7.68	-0.002	-2.11	-0.002	-0.67
Refinery Production	-0.069	-5.74	-0.063	-1.86	0.046	0.27
Stocks at sea						
User cost	-0.001	-3.07	-0.001	-1.47	-0.001	-0.15
World oil production	0.217	10.95	0.220	4.41	0.321	4.64
Non-OPEC Production						
Price	0.077	7.87	0.083	2.60	-0.010	-0.11
OPEC	-0.294	-13.80	-0.295	-5.36	-0.031	-0.08
Returns to Storage						
Stocks	0.007	0.16	-0.003	-0.01	0.111	0.94
Volatility	-0.162	-15.02	-0.160	-2.22	-0.012	-1.19
Markups						
Beginning stocks primary markets	-0.247	-10.77	-0.255	-2.43	-0.122	-0.60
Beginning stocks at sea	-0.264	-11.45	-0.271	-4.03	-0.248	-1.90
Beginning govt. stocks	-1.301	-23.55	-1.317	-7.15	-0.891	-2.21
Refinery production	0.164	5.12	0.176	2.38	0.095	0.52
Trend	0.001	1.43	0.002	0.64	0.005	0.25

stock changes, or the flow increment to final demand. Nevertheless, compared to user costs, inventories of crude oil appear more sensitive to refinery production for stocks held on land in primary markets and to world crude oil production for inventories at sea. The weighted average price elasticity of flow and stock demand is around -0.15, which is within the range of previous estimates of crude oil own price elasticities of demand estimated using annual data.

The estimates for non-OPEC production are interesting. The price elasticity of non-OPEC supply is 0.08, which supports the widely held belief that crude oil production is very rigid to price in the short-run. Our estimates also reveal that non-OPEC production moves inversely with OPEC production, suggesting that short-run production cuts by OPEC are partially offset by non-OPEC producers. This is plausible because several large non-OPEC producers, notably Norway, Mexico, and Russia, have excess capacity. Indeed, OPEC often calls on these producers to join output reductions.

As the above charts suggest, a viable returns to storage relation in the inventory to sales ratio, or days supply, is not discernable for this sample. The stocks elasticity is consistently insignificant across all three sets of estimates. On the other hand, price volatility is significant, indicating that returns to storage decline with greater price volatility, which implies that uncertainty contributes to price backwardations.

The partial equilibrium elasticities of markups with respect to the main exogenous factors in the model are also presented in Table 3. Price markups over marginal cost decrease with higher beginning inventories. For example, a 5 percent increase in beginning inventories on land or at sea, implies roughly a 1 percent decline in price. The markup elasticity with respect to government stocks is more than unity. Markups also increase with refinery production and the trend term in the net requirements equation.

The elasticities of demand for OPEC are not a simple weighted average of the market flow and stock demand elasticities because there is no closed form solution for equilibrium OPEC production, which affects non-OPEC production, which in turn affects the residual demand for OPEC crude oil. A similar feedback affects the demand for stocks held at sea, which depends upon world oil production. To determine OPEC demand elasticities, an implicit differential equation must be solved for OPEC's residual demand.[6] The estimated price elasticity of demand for OPEC oil is –2.36 at the mean and ranges from –11.65 to –1.46. Hence, OPEC is operating on the elastic portion of its residual demand curve. As a result, marginal revenue is positive, which is confirmed numerically. Computation of a complete set of market equilibrium elasticities of supply and demand involves the solution of a set of nonlinear, implicit differential equations, which lies outside the resources of this project. The following model simulations, however, essentially provide a numerical simulation of the elasticities with respect to government stock changes and supply and demand shocks.

Model Simulations

The model provides a more flexible tool for policy simulations than previous versions. To illustrate these capabilities, consider three sets of simulations that involve comparisons between a base simulation and three scenarios over a three-month period from October to December 2000:

- 30-million-barrel stock build for the SPR,
- 1-million-barrel per day supply disruption, and
- 1-million-barrel per day demand shock.

[6] See Appendix C for details.

The results appear below in Table 4. The first scenario results in a 3.5-percent increase in price, less than a $1-per-barrel increase, with a most of the stock build supplied by OPEC production. Note that this simulation assumes *ending* government stocks increase 30 million barrels per day. So the increase in price occurs as *ending* stocks rise not as beginning stocks do for the markup elasticities presented above. This finding suggests that, ceteris paribus, *the re-stocking the SPR should have minimal impacts on market prices*, particularly if it is phased in over several months.

Table 4: Dynamic model simulations of SPR build and supply and demand shocks

	30 million SPR Build			*Supply disruption of 1 million barrels per day*			*Demand shock of 1 million barrels per day*		
	Oct.	*Nov.*	*Dec.*	*Oct.*	*Nov.*	*Dec.*	*Oct.*	*Nov.*	*Dec.*
Demand primary markets	-0.96	-0.02	-0.01	-1.12	-0.05	-0.03	1.56	-0.14	-0.10
Net requirements rest of world	-0.91	-0.02	-0.01	-1.08	-0.05	-0.03	1.14	-0.14	-0.10
Stocks in primary markets	-0.21	-0.15	-0.11	-0.25	-0.18	-0.14	-1.86	-1.33	-1.00
Stocks at sea	-0.02	-0.01	-0.01	-0.37	-0.25	-0.17	-0.05	-0.04	-0.03
OPEC Output	0.54	0.01	0.01	-5.97	0.03	0.02	1.34	0.08	0.06
Non-OPEC output	0.09	0.00	0.00	2.02	0.00	0.00	0.25	0.01	0.01
First nearby price	3.44	0.06	0.04	4.07	0.17	0.12	8.84	0.50	0.35
Second nearby price	2.58	0.05	0.03	3.06	0.14	0.09	6.57	0.38	0.26
Markup	4.69	0.08	0.06	5.54	0.24	0.16	12.04	0.67	0.47

The supply disruption is simulated by including an exogenous intercept shifter in the OPEC supply relation that is calibrated to result in a one-million-barrel-per-day reduction in world crude oil production, roughly the flow counterpart of the 30 million barrel stock build. This scenario also results in a modest impact on prices, with an increase of $1.15 per barrel in equilibrium prices. Note that most of the price increases results from higher OPEC markups over marginal cost.

The simulation results for the third scenario suggests that a demand shock of 1 million barrels per day demand shock increases prices more than 12 percent, substantially more than the first two scenarios, or slightly more than $2.50 per barrel. This scenario was accomplished by simulating the model with higher refinery production in the primary market demand equation and a higher value for the trend term in the net requirements function for the rest of the world. Overall, this analysis suggests that the crude oil prices and markups are relatively more sensitive to demand shocks than to supply shocks.

Summary and Policy Implications

This paper describes the estimation of a short-run econometric model of the world market for crude oil. The model involves the simultaneous determination of flows and stocks of crude oil, using the spot price to equilibrate crude oil availability with demand for immediate processing into refined products and using the futures prices to balance current and future uses of crude oil. The econometric analysis supports several notions widely held by crude oil market and policy analysts:

- The demand for crude oil is very price inelastic,
- The supply elasticity is also price inelastic,
- Inventories respond to user costs,
- Futures prices on average respond less than proportionately to spot prices.
- Prices and OPEC markups are relatively more sensitive to demand than to supply shocks.

While user costs are significant, the elasticity of stocks with respect to user costs is relatively small. Inventories of crude oil held on land are relatively more sensitive to shifts in the production of refined petroleum products. Stocks of crude in tankers at sea move directly with world oil production, acting essentially as a floating inventory for crude oil exporters.

This study suggests that econometric modeling of world crude oil markets with high-frequency data can yield practical results that may be insightful to SPR managers. The simulations presented above indicate that *gradual accumulation of government strategic stockpiles is unlikely to cause significant upward pressures on prices*. On the other hand, this study finds only modest impacts from OPEC supply disruptions, holding all other factors constant. Indeed, our model simulations find that crude oil prices and markups are more sensitive to demand shocks than to supply shocks, suggesting that the absolute value of the price elasticity of demand is relatively larger than the corresponding price elasticity for the industry supply relation. Partial-equilibrium experiments such as these, however, rarely if ever occur under real market conditions. For example, Adelman (1982) notes that the OPEC price shocks of the 1970s and early 1980s were accompanied by sizeable inventory accumulation. While the sample used in this study is for a largely deregulated market, Adelman's general point is important. Supply and demand shocks may occur simultaneously. These observations suggest that the above model could be used to identify the separate determinants of short-run price movements using decomposition analysis similar to growth accounting studies.

Several outstanding data and model specifications issues, however, should be addressed first. The database supporting the model could be substantially improved. Incorporation of data from the International Energy Agency on OECD stocks and flows is necessary. Our data investigations uncovered estimates of monthly data on crude stocks in non-OECD countries, which deserves further analysis.

Another important issue involves corrections for first order serial correlation. The high degree of autocorrelation found in many of the equations suggests dynamic misspecification. Indeed, this is a problem for many Euler-equation based inventory models derived from discounted expected cost minimization under rational expectations, such Considine (1991). Perhaps adaptive expectations, which are rational under certain circumstances, deserve consideration. This approach certainly would be straightforward to apply to the demand equations and supply relations for the model presented above.

The above model assumes that OPEC acts as a dominant firm with a competitive fringe. While this assumption seems plausible, it should be tested against alternative imperfectly competitive market structures. This would entail disaggregating the supply relations, perhaps along the lines suggested by previous studies of OPEC behavior, in which OPEC is divided into three groups: high income, lightly populated countries such as Saudi Arabia and the Persian Gulf Emirates, medium income, higher populated states such as Iran and Algeria, and low income, densely populated countries, such as Nigeria and Indonesia. Of course, disaggregating the model would increase its complexity.

Another research need is to make refinery output and inventory determination endogenous. The study by Considine (1997) provides a basis for specifying a model of the refining sector, although the data requirements could be challenging at the world level. This extension could provide insights into how crude oil market shocks are transmitted to product markets and how these responses feedback to crude markets.

Finally, the current version of the model assumes that futures prices respond in the same way to spot prices regardless of whether the market is in backwardation or contango. Likewise, the response of inventories to user costs is also symmetric in the current version of the model. Testing for these potential asymmetries could uncover the role of these extreme observations for user costs in affecting the model parameters.

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Appendix A: Derivation of Supply Relation

Equation (13) is obtained by first solving equation (12) for the log of real price:

$$
\begin{aligned}
-\mu_1 \ln\left(\frac{P_t}{CPI_t}\right) = {} & \mu_0 + \left(\alpha_2 + \delta_2/D_t\right)\ln Z_t + \beta_2 T - \pi_2 \ln Y_t^o - Y_t^o + \gamma_2 \ln\left(Y_t^o + Y_t^{no}\right)\Big/D_t \\
& -\phi_1\left(\delta_1 + \gamma_1\right)\ln\left(X_{t-1}/Q_t\right)\big/D_t - \left(\delta_1 + \gamma_1\right) i_t / D_t \\
& -\left[\left(1-\delta_3\right)X_{t-1} + \left(1-\lambda_3\right)S_{t-1} - \left(G_t - G_{t-1}\right)\right]\Big/D_t + \varepsilon_{t-1} + v_t
\end{aligned}
$$

Multiplying both sides by -1:

$$
\begin{aligned}
\mu_1 \ln\left(\frac{P_t}{CPI_t}\right) = {} & -\mu_0 - \left(\alpha_2 + \delta_2/D_t\right)\ln Z_t - \beta_2 T + \pi_2 \ln Y_t^o + Y_t^o - \gamma_2 \ln\left(Y_t^o + Y_t^{no}\right)\Big/D_t \\
& +\phi_1\left(\delta_1 + \gamma_1\right)\ln\left(X_{t-1}/Q_t\right)\big/D_t + \left(\delta_1 + \gamma_1\right) i_t / D_t \\
& +\left[\left(1-\delta_3\right)X_{t-1} + \left(1-\lambda_3\right)S_{t-1} - \left(G_t - G_{t-1}\right)\right]\Big/D_t - \varepsilon_{t-1} - v_t
\end{aligned}
$$

Dividing by μ_1 and solving for P_t yields:

$$
P_t^e = \exp\left[\begin{array}{l}
\left\{-\mu_0 - \left(\alpha_2 + \delta_2/D_t\right)\ln Z_t - \beta_2 T + \pi_2 \ln Y_t^o + Y_t^o - \gamma_2 \ln\left(Y_t^o + Y_t^{no}\right)\Big/D_t\right. \\
+\phi_1\left(\delta_1 + \gamma_1\right)\ln\left(X_{t-1}/Q_t\right)\big/D_t + \left(\delta_1 + \gamma_1\right) i_t / D_t \\
\left.+\left[\left(1-\delta_3\right)X_{t-1} + \left(1-\lambda_3\right)S_{t-1} - \left(G_t - G_{t-1}\right)\right]\Big/D_t - \varepsilon_{t-1} - v_t\right\}\Big/\mu_1 - \ln CPI_t
\end{array}\right]
$$

For estimation, the expression for the equilibrium price (16) along with the following autoregressive terms

$$
\rho_Q \varepsilon_{t-1}^Q = \rho_Q \left\{Q_{t-1} - \left[\alpha_0 + \alpha_1 \ln\left(P_{t-1}/CPI_{t-1}\right) + \alpha_2 \ln Z_{t-1}\right]\right\}
$$

$$
\rho_R \varepsilon_{t-1}^R = \rho_R \left\{R_{t-1} - \left[\beta_0 + \beta_1 \ln\left(P_{t-1}/CPI_{t-1}\right) + \beta_2 T_{t-1}\right]\right\}
$$

$$
\rho_{no} \varepsilon_{t-1}^{no} = \rho_{no} \left\{Y_{t-1}^{no} - \left[\pi_0 + \pi_1 \ln\left(P_{t-1}/CPI_{t-1}\right) + \pi_2 \ln Y_{t-1}^o\right]\right\}
$$

$$
\rho_f \varepsilon_{t-1}^f = \rho_f \left\{\ln P_{t-1}^f - \left[\begin{array}{c}
\ln P_{t-1} + r_t + \phi_0 + \phi_1 \ln\left(X_{t-2}/Q_{t-1}\right) \\
+\phi_2 \ln V_{t-1} \ln\left(P_{t-1}/CPI_{t-1}\right)
\end{array}\right]\right\}
$$

must be included in (13) to yield:

$$P_t = MC_t^o - \frac{\left\{Y_t^o + \pi_2 - \gamma_2\left[Y_t^o/\left(Y_t^o + Y_t^{no}\right)\right]/D_t\right\}P_t^e}{\mu_1} + \varepsilon_t^p, \quad \varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + v_t^p$$

where

$$\begin{aligned} P_t^e = \exp\Big[\Big\{&-\mu_0 - \left(\alpha_2 + \delta_2/D_t\right)\ln Z_t - \beta_2 T + \pi_2 \ln Y_t^o + Y_t^o \\ &-\gamma_2 \ln\left(Y_t^o + Y_t^{no}\right)/D_t + \phi_1\left(\delta_1 + \gamma_1\right)\ln\left(X_{t-1}/Q_t\right)/D_t + \left(\delta_1 + \gamma_1\right)i_t/D_t \\ &+\left[\left(1-\delta_3\right)X_{t-1} + \left(1-\lambda_3\right)S_{t-1} - \left(G_t - G_{t-1}\right)\right]/D_t \\ &-\rho_Q\left\{Q_{t-1} - \left[\alpha_0 + \alpha_1 \ln\left(P_{t-1}/CPI_{t-1}\right) + \alpha_2 \ln Z_{t-1}\right]\right\} \\ &-\rho_R\left\{R_{t-1} - \left[\beta_0 + \beta_1 \ln\left(P_{t-1}/CPI_{t-1}\right) + \beta_2 T_{t-1}\right]\right\} \\ &+\rho_{no}\left\{Y_{t-1}^{no} - \left[\pi_0 + \pi_1 \ln\left(P_{t-1}/CPI_{t-1}\right) + \pi_2 \ln Y_{t-1}^o\right]\right\} \\ &+\left(\delta_1 + \gamma_1\right)\rho_f\left\{\ln P_{t-1}^f - \begin{bmatrix} \ln P_{t-1} + r_t + \phi_0 + \phi_1 \ln\left(X_{t-2}/Q_{t-1}\right) \\ +\phi_2 \ln V_{t-1} \ln\left(P_{t-1}/CPI_{t-1}\right) \end{bmatrix}\right\} \\ &-v_t\Big\}/\mu_1 - \ln CPI_t\Big] \end{aligned}$$

Markup elasticities are obtained by partially differentiating (16) with respect to the exogenous variables. First, consider the response of markups to beginning period inventories:

$$\frac{\partial \ln MARK_t}{\partial \ln X_{t-1}} = \frac{-\left\{Y_t^o + \pi_2 - \gamma_2\left[Y_t^o/\left(Y_t^o + Y_t^{no}\right)\right]/D_t\right\}\left[\phi_1\left(\delta_1 + \gamma_1\right) + \left(1-\delta_3\right)X_{t-1}\right]/D_t}{\mu_1^2 MARK_t}$$

Similarly the markup elasticity with respect to inventories at sea are:

$$\frac{\partial \ln MARK_t}{\partial \ln S_{t-1}} = \frac{-\left\{Y_t^o + \pi_2 - \gamma_2\left[Y_t^o/\left(Y_t^o + Y_t^{no}\right)\right]/D_t\right\}\left[\left(1-\gamma_3\right)S_{t-1}\right]/D_t}{\mu_1^2 MARK_t} .$$

The elasticity of the markup with respect to changes in beginning stocks held by governments is:

$$\frac{\partial \ln MARK_t}{\partial \ln G_{t-1}} = \frac{-\left\{Y_t^o + \pi_2 - \gamma_2 \left[Y_t^o \Big/ \left(Y_t^o + Y_t^{no}\right)\right] \Big/ D_t\right\} G_{t-1}/D_t}{\mu_1^2 MARK_t}.$$

The refinery production and time trend elasticities are:

$$\frac{\partial \ln MARK_t}{\partial \ln Z_t} = \frac{\left\{Y_t^o + \pi_2 - \gamma_2 \left[Y_t^o \Big/ \left(Y_t^o + Y_t^{no}\right)\right] \Big/ D_t\right\} \left(\alpha_2 + \delta_2/D_t\right)}{\mu_1^2 MARK_t}.$$

$$\frac{\partial \ln MARK_t}{\partial \ln T} = \frac{\left\{Y_t^o + \pi_2 - \gamma_2 \left[Y_t^o \Big/ \left(Y_t^o + Y_t^{no}\right)\right] \Big/ D_t\right\} \beta_2}{\mu_1^2 MARK_t}.$$

Appendix B: Data Sources

	Definition	***Source***
Prices		
S	First Nearby Price for WTI futures contract for crude oil	Bridge/CRB & NYMEX
F	Second Nearby Price for WTI futures contract for crude oil	Bridge/CRB & NYMEX
V	Monthly average of 20 day moving standard deviation in S	Computed based using S
Stocks		
XSEA	Stocks of crude oil at sea	EIG Data, Energy Intelligence Group
XSPR	Strategic stocks of crude oil in key reporting areas*	EIG Data, Energy Intelligence Group
XUSA	Commercial stocks of crude oil in USA	EIG Data, Energy Intelligence Group
XJAP	Commercial stocks of crude oil in Japan	EIG Data, Energy Intelligence Group
XEUN	Commercial stocks of crude oil in EU15** plus Norway	EIG Data, Energy Intelligence Group
X	Aggregate commerical stocks on land	XUSA + XJAP + XEUN - XSEA - XSPR
CRPTR	Crude and product inventories in key reporting areas	EIG Data, Energy Intelligence Group
CRSTT	Crude inventories in key reporting areas	EIG Data, Energy Intelligence Group
XP	Product stocks	CRPTR - CRSTT
XAGG	Total reported crude oil stocks	XUSA + XJAP + XEUN + XSEA + XSPR
Flows		
WPROD	World crude oil production	EIG Data, Energy Intelligence Group
Q	US+EU-15+Japan Crude Runs	EIG Data Source, Energy Intelligence Group
Y	Petroleum product demand US+EU-15+Japan	EIG Data, Energy Intelligence Group
QROW	Rest-of-world crude oil disappearance	WPROD - (XAGG - XAGG(-1)) / DAYS - Q;
Misc.		
CPI	Consumer price index in USA	U.S. Bureau of Labor Statistics
R	Three month Treasury Bill Rate	U.S. Federal Reserve Board

Appendix C: Derivation of OPEC Price Elasticity of Demand

The demand for OPEC output is as follows:

$$
\begin{aligned}
Y_t^o &= \alpha_0 + \left[\alpha_1 + \beta_1 - \pi_1 - \phi_2\left(\delta_1 + \gamma_1\right)\ln V_t\right]\ln\left(P_t / CPI_t\right) + \alpha_2 \ln Z_t + \beta_0 + \beta_2 T_t \\
&\quad - \left[\pi_0 + \pi_2 \ln Y_t^o\right] - \left[X_{t-1} + S_{t-1} - \left(G_t - G_{t-1}\right)\right] \big/ D_t \\
&\quad + \left\{\delta_0 - \delta_1\left[\phi_0 + \phi_1 \ln\left(X_{t-1}/Q_t\right) + i_t\right] + \delta_2 \ln Z_t + \delta_3 X_{t-1}\right\} \big/ D_t \\
&\quad + \left\{\gamma_0 - \gamma_1\left[\phi_0 + \phi_1 \ln\left(X_{t-1}/Q_t\right) + i_t\right] + \gamma_2 \ln\left(Y_t^o + Y_t^{no}\right) + \gamma_3 S_{t-1}\right\} \big/ D_t + \varepsilon_{t-1} + v_t,
\end{aligned}
$$

Notice that a closed form solution for OPEC output is not possible:

$$
\begin{aligned}
Y_t^o + \pi_2 \ln Y_t^o - \gamma_2 \ln\left(Y_t^o + Y_t^{no}\right) \big/ D_t &= \alpha_0 + \left[\alpha_1 + \beta_1 - \pi_1 - \phi_2\left(\delta_1 + \gamma_1\right)\ln V_t\right]\ln\left(P_t / CPI_t\right) \\
&\quad + \alpha_2 \ln Z_t + \beta_0 + \beta_2 T_t - \pi_0 - \left[X_{t-1} + S_{t-1} - \left(G_t - G_{t-1}\right)\right] \big/ D_t \\
&\quad + \left\{\delta_0 - \delta_1\left[\phi_0 + \phi_1 \ln\left(X_{t-1}/Q_t\right) + i_t\right] + \delta_2 \ln Z_t + \delta_3 X_{t-1}\right\} \big/ D_t \\
&\quad + \left\{\gamma_0 - \gamma_1\left[\phi_0 + \phi_1 \ln\left(X_{t-1}/Q_t\right) + i_t\right] + \gamma_3 S_{t-1}\right\} \big/ D_t + \varepsilon_{t-1} + v_t,
\end{aligned}
$$

Consider, however, the differential of this equation, holding all other factors other than price and non-OPEC production constant:

$$
dY_t^o + \pi_2 \frac{dY_t^o}{Y_t^o} - \gamma_2 \frac{dY_t^o + dY_t^{no}}{\left(Y_t^o + Y_t^{no}\right) D_t} = \alpha_0 + \left[\alpha_1 + \beta_1 - \pi_1 - \phi_2\left(\delta_1 + \gamma_1\right)\ln V_t\right] d\ln\left(P_t / CPI_t\right),
$$

which is equivalent to:

$$
d\ln Y_t^o \left[Y_t^o + \pi_2 - \frac{Y_t^o \gamma_2}{\left(Y_t^o + Y_t^{no}\right) D_t} \right] = \left[\begin{array}{c} \alpha_1 + \beta_1 - \pi_1 \\ -\phi_2\left(\delta_1 + \gamma_1\right)\ln V_t \end{array} \right] d\ln\left(P_t / CPI_t\right) + \frac{Y_t^{no} \gamma_2}{\left(Y_t^o + Y_t^{no}\right) D_t} d\ln Y_t^{no}
$$

The price elasticity of OPEC demand is as follows:

$$
\frac{d\ln Y_t^o}{d\ln P_t} = \left[\alpha_1 + \beta_1 - \pi_1 - \phi_2\left(\delta_1 + \gamma_1\right)\ln V_t\right] \left[Y_t^o + \pi_2 - \frac{Y_t^o \gamma_2}{\left(Y_t^o + Y_t^{no}\right) D_t} \right]^{-1}
$$

The inverse of this last expression is equal to the inverse price elasticity of demand evaluated at equilibrium price and quantity.